



# Exchange programme Vrije Universiteit

Vrije Universiteit Amsterdam - Exchange programme Vrije Universiteit - 2022-2023

## Exchange

Vrije Universiteit Amsterdam offers many English-taught courses in a variety of subjects, ranging from arts & culture and social sciences, neurosciences and computer science, to economics and business administration.

The International Office is responsible for course approval and course registration for exchange students. For details about course registration, requirements, credits, semesters and so on, please [visit the exchange programmes webpages](#).

# Mathematical Analysis

Course Code	XB_0009
Credits	6.00
Period	P4+5
Course Level	100
Language Of Tuition	English
Faculty	Faculty of Science
Course Coordinator	G. Benedetti
Examiner	G. Benedetti
Teaching Staff	G. Benedetti
Teaching method(s)	Partial Exam, Seminar, Lecture

## Course Objective

After this course, the student...

1. ... knows basic definitions concerning limits and continuity (convergence, Cauchy sequence, limit, completeness, continuity, uniform continuity) and is able to determine whether a sequence, series or function satisfies these definitions;
2. ... knows the definition of differentiability (i.e., that a function can be approximated by a linear one) and can determine whether a function (and in particular, a power series) is differentiable;
3. ... knows the definition of Riemann integrability and can prove that certain functions (in particular, polynomials, monotone and uniformly continuous functions) are Riemann integrable;
4. ... knows the definition of basic concepts from metric topology (metric, convergence, completeness, Banach space) and can prove that simple examples satisfy these definitions;
5. ... knows the statement of the Banach Fixed Point Theorem, and can apply this theorem to solve fixed point equations (in particular integral and differential equations).

## Course Content

This course treats the rigorous mathematical theory behind Calculus: limits, continuity, linear approximation, differentiability, integrability, and the mutual relation between these concepts. The mathematical tools that are necessary for formulating and proving the essential results of Calculus are first presented in the context of real valued sequences and real valued functions of a real variable, in such a way that everything can later be generalised (to  $Y$ -valued functions of variables in  $X$ , with  $X$  and  $Y$  Banach spaces). The space  $C[a,b]$  of real valued continuous functions on an interval  $[a,b]$  will appear as the first example of such a Banach space.

Starting point of the course are an ancient iterative scheme for solving equations, and the fundamental properties of (the set of) real numbers. Highlights: a fairly complete exposition of power series directly based on a systematic algebraic approach for monomials, and an early introduction of the Implicit Function Theorem via a contraction argument and the Banach Fixed Point Theorem.

Topics covered:

1. Cauchy sequences, convergence, limits;
2. Completeness of the real numbers; theorem of Bolzano-Weierstrass;
3. Continuity and uniform continuity;
4. The concept of differentiability (including differentiability of power series);
5. The concept of Riemann integrability (including Riemann integrability

of monotone and uniformly continuous functions);  
6. The language of metric topology;  
7. Completeness of the space  $C[a,b]$ ; uniform convergence;  
8. The Banach Fixed Point Theorem (with applications to integral and differential equations, and the implicit function theorem).

## Additional Information Teaching Methods

Lectures, study sessions and tutorials (2+1+2 hours per week).

You are also required to hand in a homework assignment every other week.

We expect you to dedicate in total about 10 hours per week to this course.

## Method of Assessment

Your final grade is built up as follows:

A written midterm exam [40%];  
A written final exam [50%];  
Six written assignments [10%].

To pass the course...

... your final score must be no less than 55% (all students);  
... you must have been present at 70% of the study sessions and tutorials (full time students only);

If you don't fulfil these requirements, then you must take the resit exam. The resit exam then counts for 100% (i.e. the partial grades that you obtained during the course will no longer be valid).

## Entry Requirements

Basic Concepts in Mathematics (or another course on general mathematical language, notation, and concepts, including proof by induction and elementary combinatorics);

## Literature

Course notes will be made available through Canvas.

## Additional Information Target Audience

Bachelor Mathematics Year 1

## Additional Information

Participation in 70% of study sessions and tutorials is mandatory (for full time students) in order to pass the course.

## Recommended background knowledge

Single Variable Calculus